

RG analysis and Magnetic instability in gapless superconductors

Deog Ki Hong^{1,2,*}

¹*Department of Physics, Pusan National University, Busan 609-735, Korea*

²*Center for Theoretical Physics, Massachusetts Institute of Technology,
Cambridge, MA 02139, USA*

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Abstract

We study the magnetic instability of gapless superconductors. The instability arises due to the infrared divergence associated with the gapless modes of the superconductor. When the Fermi-surface mismatch between pairing fermions is close to the gap, the gapless modes have a quadratic energy dispersion relation at low energy and open a secondary gap at the Fermi surface, which is only power-suppressed by the coupling. On the other hand, for a large mismatch, we find the gapless superconductor does not open a secondary gap, but instead makes transition to a new phase by forming the condensate of supercurrents. We calculate the condensate of supercurrents by minimizing the effective potential. In the new phase, the Meissner mass is positive for the magnetic fields orthogonal to the direction of the condensate but zero in the parallel direction.

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*E-mail: dkhong@pusan.ac.kr, dkhong@lms.mit.edu

How matter behaves at extreme density is ultimately related to the fundamental questions on basic building blocks of matter. Quantum chromodynamics (QCD), which is believed to be the theory of strong interaction of elementary particles, predicts matter at extreme density is color superconducting quark matter, where the quarks form Cooper-pairs [1]. The search for quark matter in heavy ion collision or in compact stars is currently under intense investigation.

Quark matter is expected to exhibit a rich phase structure, having various pairing patterns for quarks, as temperature or density changes. At high density, where the SU(3) flavor symmetry among u, d, s quarks is good, the QCD interaction pairs quarks to form color-flavor-locked (CFL) matter, respecting the flavor symmetry [2]. The flavor-asymmetric electroweak interaction and quark mass strain the CFL matter, which then becomes unstable under a large stress.

BCS pairing breaks in general when the stress is bigger than the pairing energy or $\delta\mu > \Delta_0/\sqrt{2}$, where the stress $2\delta\mu$ is the chemical potential difference of pairing quarks and Δ_0 is the BCS gap [3]. Recently, however, it has been shown that Cooper-pairing may be stable even if the stress is bigger than the gap, known as Sarma phase [4], if one enforces the electric neutrality in quark matter [5] or uses a momentum-dependent kernel for pairing [6]. The salient feature of such asymmetric quark matter is that quarks can be excited at arbitrarily low energy, though it is superconducting. The gapless superconductors, if realized, will be a new kind of Fermi liquid, which has both properties of the usual BCS superconductor and the Landau-Fermi liquid. It was soon found, however, that the gapless superconductors have negative Meissner mass squared [7] or a negative supercurrent density [8], showing magnetic instability of the system.

In this letter we calculate the effective potential for the gapless superconductors and study the magnetic instability. We find that the magnetic instability arises because of the infrared divergences associated with the gapless modes. When the stress is greater than the gap, $\delta\mu \gtrsim \Delta$, the infrared divergence is logarithmic and the system develops a condensate of supercurrents. On the other hand, when the stress is very close to the gap, $\delta\mu \approx \Delta$, the divergence is much more severe and it inevitably leads to opening a secondary gap at the Fermi surface.

To study the secondary gap in detail, we derive the low energy effective Lagrangian for the gapless modes by integrating out the gapped modes, which turns out to have an attrac-

tive four-Fermi interaction. Unlike the ordinary Fermi liquid, the four-Fermi interaction in gapless superconductors scales like $s^{-1/2}$ as one scales down to the Fermi surface ($s \rightarrow 0^+$), if incoming fermions have equal and opposite momenta [10], and induces a gap which is only power-suppressed in couplings. Both renormalization group (RG) analysis and gap equation analysis show that the gapless excitations develop a gap, which naturally stabilizes the system.

Let us consider a minimal model for gapless superconductivity, which has two different flavors that pair, denoted as up and down, and free electrons for electric neutrality, shown in Table I. The pairing force is $SU(2)_c$ color interaction under which the up and down particles are fundamental. The system is not a color superconductor, since the Cooper pair is a color-

	chemical potential	electric charge	$SU(2)_c$
ψ_1 (up)	$\mu_1 = \bar{\mu} - \delta\mu$	$q_1 = \bar{q} + \delta q$	2
ψ_2 (down)	$\mu_2 = \bar{\mu} + \delta\mu$	$q_2 = \bar{q} - \delta q$	2
ψ_e (electron)	$\mu_e = 2\delta\mu$	$q_e = -1$	1

TABLE I: The electron chemical potential is $\mu_e = \mu_2 - \mu_1$ and $q_e = -2\delta q$, since the system is in equilibrium under the weak interaction, $\psi_2 \leftrightarrow \psi_1 + e^- + \bar{\nu}_e$.

singlet. However, it is an electric superconductor and exhibits all the essential features of gapless superconductivity, including the magnetic instability. It is straightforward to extend to the gapless 2-flavor color-superconductor (g2SC) or to the gapless CFL superconductor (gCFL). Neglecting the anti-particles, the system is described by a Lagrangian density

$$\mathcal{L} = \sum_{i=1,2} \psi_i^\dagger [i\partial_t - E(\vec{p}) + \mu_i] \psi_i + \frac{G}{2} \epsilon^{ij} \psi_i^\dagger \psi_j^\dagger \epsilon^{i'j'} \psi_{i'} \psi_{j'} + \mathcal{L}'_{\text{int}}, \quad (1)$$

where $E(\vec{p})$ is a flavor-independent function of momentum and the color and spin indices are suppressed. Gluons are integrated out to generate an attractive four-Fermi interaction, antisymmetric in color and flavor, and also other irrelevant interactions, denoted as $\mathcal{L}'_{\text{int}}$, assuming the gluon exchange interaction is most attractive for quarks antisymmetric in color and flavor.

Neglecting the irrelevant interactions, one can calculate the free energy in the mean field approximation,

$$\Omega_s(\Delta, \delta\mu) = \frac{\Delta^2}{G} - 2 \int \frac{d^3p}{(2\pi)^3} \left[\sqrt{\epsilon^2 + \Delta^2} + \delta\mu + \left| \sqrt{\epsilon^2 + \Delta^2} - \delta\mu \right| \right] - \frac{(2\delta\mu)^4}{12\pi^2}, \quad (2)$$

where $\epsilon(p) = E(p) - \bar{\mu}$ and the last term is the electron free energy. When $\delta\mu > \Delta$, the gap equation has an unstable solution, $\Delta = \sqrt{\Delta_0(2\delta\mu - \Delta_0)}$, where Δ_0 is the gap when $\delta\mu = 0$. This unstable Sarma phase can be stabilized if charge neutrality is enforced. The electric charge neutrality condition for the Sarma phase, $\langle Q_{\text{em}} \rangle = 0$, is satisfied for $\bar{\mu} \gg \delta\mu$ if $\delta\mu = \sqrt{\delta\mu_0^2 + \Delta^2}$, where $2\delta\mu_0$ is the chemical potential difference of the free neutral system [9]. Compared with the free neutral system, the free energy becomes for $\bar{\mu} \gg \delta\mu$

$$\Omega_s(\Delta, \delta\mu) - \Omega_{\text{free}} \simeq -\frac{\bar{\mu}^2}{\pi^2} \delta\mu_0^2 \left[1 - \left(\frac{\Delta_0}{2\delta\mu_0} - 1 \right)^2 \right]. \quad (3)$$

The neutrality enforcement stabilizes the Sarma phase if $\Delta_0/4 \lesssim \delta\mu_0 \lesssim \Delta_0/2$ or the gap $0 \lesssim \Delta \lesssim \Delta_0/\sqrt{2}$. At the minimal $\delta\mu_0$, the ratio $\delta\mu/\Delta$ is smallest and close to $7/4 - 1/\sqrt{2}$.

We now calculate the Coleman-Weinberg effective potential [11] for photon fields in the Sarma phase, which will be same as the free energy in Eq. (2) except that $p_0 \rightarrow p_0 + q_i e A_0$ and $\vec{p} \rightarrow \vec{p} + q_i e \vec{A}$ [12]. As we integrate out the high frequency modes, the effective potential will get non-local terms, which can be expanded in powers of momentum. When all the modes with $\omega \gtrsim \delta\mu$ are integrated out, only the gapless modes survive and the effective potential becomes

$$V(A) = -4 \int_{\Lambda} \frac{d^4 p_E}{(2\pi)^4} \ln \left[p_4^2 + \left(\sqrt{\bar{\epsilon}_1 \bar{\epsilon}_2 + \Delta^2} - \delta\bar{\mu} \right)^2 \right] + C_0(\Lambda) + \frac{C_2(\Lambda)}{2} \vec{A}^2 - \frac{B_2(\Lambda)}{2} A_0^2 + \dots \quad (4)$$

where $\delta\bar{\mu} = \delta\mu - \delta q e A_0$ and $\bar{\epsilon}_i = E(\vec{p} + q_i e \vec{A}) - \bar{\mu} - \bar{q} e A_0$. In the effective potential we have introduced an ultra-violet cutoff Λ ($\lesssim \delta\mu$) and the counter-terms, C_i 's and B_i 's. The ellipsis denotes the higher order terms in photon fields and their derivatives.

As we will see, the gapless superconductor suffers instability due to the infrared divergences associated with the gapless quasiparticle modes. Depending on how close the stress is to the gap, the structure of divergences and their physical consequences differ drastically. When the stress is not too close to the gap ($\delta\mu \gtrsim \Delta$), as in g2SC, the quasi-particles have approximately linear dispersion relation near the Fermi surface. In this case there is a logarithmic infrared divergence and the system is stabilized by spontaneously generating Goldstone currents. However, when the stress is close to the gap ($\delta\mu \approx \Delta$), as in gCFL, the quasiparticles near the Fermi surface have approximately quadratic dispersion relation and there is genuine instability in the system, which leads to opening a secondary gap at the Fermi surface.

We consider first the case where the stress is not too close to the gap. If we integrate out further the quasi-particle modes till the remaining has an approximately linear dispersion relation, the effective potential becomes

$$V(A) = -\nu_1 \eta^{-1} \int \frac{d\Omega_{\vec{v}_1}}{4\pi} (eV_1 \cdot A)^2 \left[\ln \frac{\Lambda^2}{(eV_1 \cdot A)^2} + 1 \right] + (1 \rightarrow 2) \quad (5)$$

$$+ C_0(\Lambda) + \frac{C_2(\Lambda)}{2} \vec{A}^2 - \frac{B_2(\Lambda)}{2} A_0^2 + \dots$$

where ν_1 and ν_2 are the density of states at the Fermi surfaces at p_1 and p_2 , respectively. At the inner and outer Fermi surfaces, $\epsilon(p_1) = -\sqrt{\delta\mu^2 - \Delta^2}$ and $\epsilon(p_2) = \sqrt{\delta\mu^2 - \Delta^2}$. The angular integrations are over the inner and outer Fermi velocities, $\vec{v}_1 = \partial E / \partial \vec{p}_1$ and $\vec{v}_2 = \partial E / \partial \vec{p}_2$. We have also defined $\eta = \delta\mu / \sqrt{\delta\mu^2 - \Delta^2}$, $V_1^\mu = (\eta\bar{q} + \delta q, \eta\bar{q} \vec{v}_1)$, and $V_2^\mu = (\eta\bar{q} - \delta q, \eta\bar{q} \vec{v}_2)$.

We see that the one-loop effective potential due to the gapless modes is negative for both A_0 and \vec{A} , and its second derivative is negative infinity at the origin. Since the Meissner mass term in the potential has the positive sign, while the Debye mass has a wrong sign, the effective potential has a minimum away from the origin for \vec{A} . We fix the counter-terms by imposing the renormalization conditions at a scale M ,

$$\left. \frac{\partial^2 V}{\partial A_0^2} \right|_{A_0=M} = -m_D^2, \quad \left. \frac{1}{3} \delta_{ij} \frac{\partial^2 V}{\partial A_i \partial A_j} \right|_{\vec{A}^2=M^2} = m_M^2, \quad (6)$$

to get the effective potential

$$V(A) = \frac{1}{2} m_M^2 \vec{A}^2 - \frac{\eta}{3} (\nu_1 v_1^2 + \nu_2 v_2^2) e^2 \bar{q}^2 \vec{A}^2 \left[\ln \left(\frac{M^2}{\vec{A}^2} \right) + 3 \right]$$

$$- \frac{1}{2} m_D^2 A_0^2 - \frac{1}{\eta} (\nu_1 + \nu_2) (\eta\bar{q} + \delta q)^2 e^2 A_0^2 \left[\ln \left(\frac{M^2}{A_0^2} \right) + 3 \right] \quad (7)$$

The physics lies in the renormalization conditions that we imposed in Eq. (6). If we take $M = 0$, we will recover the negative mass squared for the Meissner mass, similar to the result obtained in [13]. However, this result is sensitive to the ultraviolet cutoff. Here instead we take a nonzero M such that the Meissner mass due to modes with $\omega > M$ is nonnegative and then we study how the system flows as we change M , keeping the ultraviolet physics in the renormalization conditions.

The vector fields in Eq. (7) are not physical, because the effective potential we derived is not manifestly gauge-invariant. For the physical degrees of freedom, we go to a unitary gauge, where the vector fields are replaced by the gauge-covariant combination of gauge

fields and Goldstone fields, $\vec{A} \rightarrow \vec{A} - \vec{\nabla}\varphi$, $A_0 \rightarrow A_0 + \partial_t\varphi$. By minimizing the effective potential, Eq. (7), we find that the system develops a condensate, breaking the rotational invariance,

$$\left\langle \left(\vec{A} - \vec{\nabla}\varphi \right)^2 \right\rangle = M^2 \exp \left[2 - \frac{3m_M^2}{2\eta(\nu_1 v_1^2 + \nu_2 v_2^2) e^2 \bar{q}^2} \right]. \quad (8)$$

Since the condensate is invariant under the renormalization group flow, we calculate it at $M = \delta\mu$. The Meissner mass due to the fast modes ($\omega \geq \delta\mu$) is found to be $m_M^2 \simeq (8/3) \bar{q}^2 e^2 \nu_* v_*^2$, where ν_* and v_* are the density of states and the quasiparticle velocity at the pairing momentum, p_* , respectively. We find

$$\left\langle \left(\vec{A} - \vec{\nabla}\varphi \right)^2 \right\rangle \simeq \delta\mu^2 \exp \left[2 - \frac{4\nu_* v_*^2}{\eta(\nu_1 v_1^2 + \nu_2 v_2^2)} \right], \quad (9)$$

which shows the system has a spontaneously generated Goldstone current even in the absence of external gauge fields, $\langle -\vec{\nabla}\varphi \rangle \equiv \vec{A}_c \neq 0$ [14]. The ground state, however, does not carry any net current, since $\langle \vec{J} \rangle \equiv -\partial V(A)/\partial \vec{A} \big|_{\vec{A}=\vec{A}_c} = 0$ [15].

The gapless superconductor is a directionally perfect diamagnet because the effective potential has flat directions along the angular rotation of \vec{A}_c . Under an external field \vec{A} , the current is given as

$$J_i(x) = - \frac{\partial^2 V(A)}{\partial A_i \partial A_j} \bigg|_{\vec{A}=\vec{A}_c} (A_j - \partial_j \varphi). \quad (10)$$

Taking the curl of the current, we get $\nabla^2 \vec{B} = c_M^2 \left(\vec{B} - \vec{B} \cdot \hat{A}_c \hat{A}_c \right)$, where \hat{A}_c is the unit vector along the condensate and $c_M^2 = \frac{2\eta}{3} (\nu_1 v_1^2 + \nu_2 v_2^2) e^2 \bar{q}^2$. The Meissner mass is c_M for the magnetic fields orthogonal to the condensate and zero for the parallel fields.

When $\delta\mu \approx \Delta$, $p_1 \approx p_2 \approx p_*$ and the gapless modes have a quadratic dispersion relation $\omega(\vec{p}) \simeq (\vec{v}_* \cdot \vec{l})^2 / (2\delta\mu)$ for $0 \lesssim \omega \lesssim \delta\mu$, where the residual momentum $\vec{l} = \vec{p} - \vec{p}_*$. The effective potential due to the quadratic gapless modes is found to be, with $V_* = (1, \vec{v}_*)$,

$$\delta V(A) = - \frac{16\Gamma(5/4)\Gamma(1/4)}{3\sqrt{\pi}} \int \frac{d\Omega_{\vec{v}_*}}{4\pi} \frac{\nu_* |e|^3}{2\delta\mu |\vec{v}_*|} |q_1 V_* \cdot A - \delta q A_0|^{3/2} |q_2 V_* \cdot A + \delta q A_0|^{3/2}. \quad (11)$$

The effective potential shows a genuine instability, not stabilized by the condensate of Goldstone currents, since it is unbounded from below. However, as we will see later, by the Kohn-Luttinger theorem [16], the quadratic gapless modes pair among themselves to open a gap at $p = p_*$ and the system becomes stable.

The Sarma phase has a gapless excitation, denoted as Ψ , whose energy spectrum is given as $\omega = \pm \left(\delta\mu - \sqrt{\epsilon(p)^2 + \Delta^2} \right)$, and a gapped excitation, denoted as Ψ_H , with energy

spectrum $\omega = \pm \left(\delta\mu + \sqrt{\epsilon(p)^2 + \Delta^2} \right)$. The energy of the gapless mode, measured from the Fermi surface is quadratic in the residual momentum except very near the Fermi surface.

Subtracting out the Fermi momentum as in the high density effective theory [17], we may write the effective Lagrangian for gapless modes as, when $\delta\mu > \omega \gtrsim \omega_{\text{IR}} \equiv (\delta\mu^2 - \Delta^2)/(2\delta\mu)$,

$$\mathcal{L}_{\text{eff}} = \sum_{\vec{v}_*} \Psi^\dagger \left[i\partial_t + \frac{(\vec{v}_* \cdot \vec{\nabla})^2}{2\delta\mu} \right] \Psi(\vec{v}_*, x) + \frac{\kappa}{2} \Psi^\dagger \Psi^\dagger \Psi \Psi + \dots, \quad (12)$$

where the summation is over all the patches that cover the Fermi surface and the ellipsis denotes the higher order interactions.

Because of the quadratic dispersion relation, as we scale down to the Fermi surface $\omega \rightarrow s\omega$ ($0 < s < 1$), the momentum parallel to the Fermi velocity \vec{v}_* scales as $\vec{v}_* \cdot \vec{l} \rightarrow s^{1/2} \vec{v}_* \cdot \vec{l}$, while the perpendicular momentum $\vec{l}_\perp = \vec{l} - \hat{v}_* \hat{v}_* \cdot \vec{l}$ does not scale. Since the action for the kinetic term has to be scale-invariant, the gapless mode scales in the momentum space as $\Psi(t, \vec{l}) \rightarrow s^{-1/4} \Psi(t, \vec{l})$. The immediate consequence of such unusual scaling is that the four-Fermi interaction becomes a relevant operator when the incoming fermions have equal and opposite Fermi momenta. Under the scale transformation the four-Fermi coupling transforms as $\kappa \rightarrow s^{-1/2} \kappa$ and it will hit an infrared singularity unless a gap opens in the infrared region.

Among the irrelevant interactions in $\mathcal{L}'_{\text{int}}$, which we have neglected so far, there is a repulsive four-Fermi interaction, $\mathcal{L}'_{\text{int}} \ni G_s \psi_1^\dagger \psi_1 \psi_2^\dagger \psi_2$, which is symmetric in color. This interaction describes transition between the gapless mode and the gapped mode and induces an attractive four-Fermi interaction for the gapless modes. In our minimal model, the attractive channel turns out to be a spin-1 and color-singlet channel and the condensate takes $\langle \Psi i\sigma_2 \sigma^i \lambda^2 \Psi \rangle \sim \Delta_s \delta^{i3}$, similar to the polar phase [18], where σ 's are the Pauli matrices in the spin space and λ^2 is the antisymmetric Pauli matrix in the color space. By integrating out the gapped modes, we have $\kappa = (\pi/4) G_s^2 \nu_*/v_*$ at one-loop (See Fig. 1).

The Cooper-pair gap equation is given as

$$\Delta_s = \kappa \int \frac{d^4 l}{(2\pi)^4} \frac{\Delta_s}{l_4^2 + \left[\frac{(\vec{l} \cdot \vec{v}_*)^2}{2\delta\mu} \right]^2 + \Delta_s^2}. \quad (13)$$

Upon integration, we get

$$\Delta_s \simeq 6.85 \kappa^2 \left(\frac{\nu_*}{v_*} \right)^2 \delta\mu = 4.2 \left(\frac{G_s}{G} \right)^2 g^4 \delta\mu, \quad (14)$$

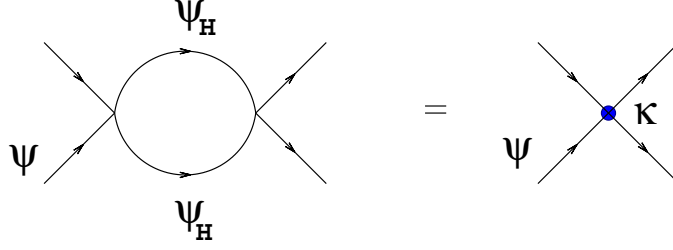


FIG. 1: The four-Fermi interaction of the gapless modes Ψ , induced by the gapped modes Ψ_H .

where $g^{-1} \equiv \ln(2\bar{\mu}/\Delta)$. For a weak coupling, $g \ll 1$, the secondary gap is well separated from the ultraviolet scale. For a strong coupling, however, one needs to go beyond the mean field approximation to find the correct secondary gap. In any case, the gapless superconductors are stabilized by opening a secondary gap at the Fermi surface if the secondary gap is bigger than the infrared cutoff of the effective theory, $\omega_{\text{IR}} \simeq (\delta\mu^2 - \Delta^2)/(2\delta\mu)$.

We have studied a minimal model for gapless superconductivity and found that the gapless superconductors necessarily open a secondary gap at the Fermi surface if the characteristic scale of the gapless superconductors ω_{IR} is small enough compared to the secondary gap. However, if $\omega_{\text{IR}} > \Delta_s$, the gapless superconductors do not open a gap but instead develop a condensate of supercurrents.

Though our analysis is generic and applies to any gapless superconductors or superfluids, the specific form of the secondary gap depends on the details of the gapless superconductors. For instance, for g2SC the gapless modes are degenerate in color and spin. Thus, if the secondary gap forms, it will be a color-antitriplet and spin-1 gap, but not exponentially suppressed. For gCFL the gap has to open in the color-sextet channel, since the gapless modes are not degenerate in color. The possible candidate is a color-sextet condensate, $\langle \Psi_L C \gamma_0 \gamma_5 \Psi_R \rangle$ [19].

To conclude, we have shown that the gapless superconductors are unstable due to the infrared divergence associated with the gapless modes and make phase transition either to superconductors with a supercurrent condensate, or to gapful superconductors. The Meissner mass is nonnegative in both cases. In the former case, the rotational symmetry is spontaneously broken and the Meissner mass is directional. In the latter case the secondary gap is not exponentially suppressed but only power-suppressed in couplings, which may have significance in neutron stars or in atomic superfluids.

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